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## **Projection of Total and Sub-Regional Population: A New Technique**

### **Introduction**

SEVERAL excellent methods have been developed for prediction as well as projection of population by sex, age, labour force participation etc. in demographic literature. Yet, the subject continues to merit attention particularly on account of increasing concern with population growth and change. A new method in this field, therefore, should have distinct advantages over existing methods even when they applied with cosmetic tinkering.

For regional projection the component method is suitable and logical in substance. A good review of the accuracy of estimates in addition to his own contributions can be found in Cohen (1986). In spite of satisfactory quality projection at least for the totals, obtained through component method (Keyfitz, 1981), the approach poses problems for the developing countries. In such countries even the current levels of fertility and mortality are not known adequately. So the hope of hitting the right combination of fertility and mortality for future is not high. The matter complicates further due to sex preference for characterising fertility, official policies like "one child" family or incentives to increase fertility.

Pattern of fertility change is not always predictable. For example, in majority of the Indian States fertility was declining between 1971-81, but during 1981-1985 it rose in both rural and urban areas of several States. Both CBR and ASFR increased and the increases were higher in the urban areas (Mukerji, 1986). In almost all the States, ASFR in the age group 20 to 24 increased significantly, and sharply declined in the age group 35 to 39. The increase

varied by states and age groups. In 1981, this could not have been anticipated. Mortality also showed unexpected pattern of change. Between 1971 and 1981 CDR and its major component IMR, remained practically stable at fairly high levels, but during 1981-85 IMR fell by almost 5 points annually in some of the States where it was high to begin with. This happened without any additional investment in the health programme or radical changes in the health delivery system. The third component of population change, namely, migration is almost unpredictable. Thus, even for short period of time, for which component method is fairly reliable, serious errors may occur in decomposing population change.

For sub-regional projection the ratio method is found to be very useful. A good account of its application with American data can be found in Pittenger (1976). Inputs for this method are less extensive than in the component method, however, some arbitrariness enters as a pattern for the "ratios" for the future years has to be chosen. In developing countries boundaries of sub-regions change due to a variety of reasons; for example, in India State and district level boundaries have kept on changing right from 1901, and it is not easy to predict how a particular ratio will behave in relation to other ratios.

Keeping these points in view, a method of projection may be developed with at least the following properties :

- (1) Input requirement should be past population data only, preferably for not more than 3 previous censuses.
- (2) The estimator should have asymptotic property independent of the fertility and mortality changes.
- (3) Projection for each sub-group should be mathematically related with the other sub-groups and with the total.
- (4) The estimated population should be accompanied by standard deviation.

Such a procedure is feasible, and it is particularly useful for projecting the sub-group populations when the total is known. The sub-group can be a State, a district, a village or an age group. Even the total population can be projected without specification of future fertility and mortality patterns; all that is necessary is the assumption that population will stop growing at a specified time point in the future. As the approach is unconventional it will be illustrated as follows :

(A) Assuming that the total population of India in 1981 and state level populations are known for the years 1951, 1961, 1971, what will be the expected populations of the States in 1981.

(B) Assuming that the total population of India is known for the years 1951, 1961, 1971 and 1981, what will be the age distribution in 1981, given the age distributions for 1951, 1961 and 1971 ?

(C) Assuming that the total population of Kerala is known for the year

1951, 1961, 1971 and 1981, what will be the population of each district in 1981 given, the district level populations for 1951, 1961 and 1971?

As the enumerated population in 1981 is known in each case the quality of projection can be examined. In India, country and State level projections are made by the component method. This is possible due to the availability of reliable vital rates from the Sample Registration Scheme, which now covers nearly 6 million persons in the States of India. For testing district level projection the State of Kerala has been chosen mainly because this State has shown very different characteristics in 1971-1981. Kerala is considered as a demographically advanced State in India (Zachariah, 1983). It has experienced sharp reductions in both fertility and mortality. The age at marriage for females is the highest among the states of India, even though the singulate mean age at marriage in some of the northern districts of Kerala was not better than some of the districts of Uttar Pradesh in north India till the year 1971. Uttar Pradesh is considered to be a problem State in India as it has low age at marriage for females, and fairly high level of fertility and mortality. Of course, from the point of view of natural growth Uttar Pradesh was never rated as a fast growing State for almost 100 years (Gupta, 1985) In addition to peculiarities in the basic demographic indicators, Kerala also experienced heavy out-migration to Gulf countries in 1971-81. It is believed that between 0.5 to 1 million persons left Kerala for jobs in Arab countries. Such migrations affected different districts differently and the flow of hard currency affected all social and demographic variables significantly. Thus, if under these demographically difficult situation the projected populations agree with the enumerated population by district in 1981 then the method should work well for other areas where the components of population change are more stable.

In applying this alternative approach, it is assumed that total population for each years 1901 to 1981 are known, and the population will stop growing between the years 2041 and 2051. No other input is necessary for estimating the future total populations.

The advantage is apparent, namely, if the total population at a future date is known through any approach then all sub-groups are estimable using only past enumerated populations and the future total population. In a sense this should be the ideal situation as the input requirement is not more than the output. If we are searching for future population by region, sex or age; the input should not be more than that and there should be no need to know the levels and paths of fertility and mortality changes.

## 2. Methodology

Estimation is based on a set of three identities which can be written as follows:

$$(1 - X) R_1 + X R_2 = R_8$$

$$(1 - X) \text{ Est } R_1 + X \text{ Est } R_2 = \text{ Est } R_s$$

$$\text{and } (1 - X) L_1 + X L_2 = L_s$$

Where,

$$R_1 = \frac{\text{Cumulated population of a sub-group upto a particular census}}{\text{Cumulated population of the same sub-group upto the previous census}}$$

$$R_2 = \frac{\text{Cumulated population of the remaining sub-groups upto a particular census}}{\text{Cumulated population of the remaining sub-groups upto the previous census}}$$

$$R_s = \frac{\text{Cumulated total population upto a particular census}}{\text{Cumulated total population upto the previous census}}$$

Est  $R_1$ , Est  $R_2$  and Est  $R_s$  are respectively equal to  $R_1$ ,  $R_2$  and  $R_s$  when there was no change in the population of each group between the previous census and the current census for which the parameters are being computed. And  $(1 - X)$  is equal to the ratio of the cumulated population of the sub-group upto the previous census to the cumulated total population upto that census.  $L_1$ ,  $L_2$  and  $L_s$  denote the differences  $(R_1 - \text{Est } R_1)$ ,  $(R_2 - \text{Est } R_2)$  and  $(R_s - \text{Est } R_s)$  respectively. Positive "L" will denote faster growth of population between the previous and current censuses. Negative "L" will indicate decline of population between the previous and the present censuses, and  $L = 0$  will indicate no growth between the previous and the present censuses. It is apparent that each  $R$  and Est  $R$  will tend to unity as time increases and each "L" will tend to zero.

If it is assumed that the total population for the year is known and we are interested in estimating the sub-group populations, then  $R_s$ , the three Est  $R$  and  $X$  are known. The unknowns are  $R_1$ ,  $R_2$ ,  $L_1$  and  $L_2$ . That is, we have four unknowns in three identities. There are two more sets of relations, namely, the sum of  $(1 - X)R_1 - R_2$  sum of  $(1 - X)L_1 = L_s$  and sum of  $(1 - X) \text{ Est } R_1 = \text{ Est } R_s$  Similarly, sum of  $X R_2 = (T - 1)R_s$  sum of  $X \text{ Est } R_2 = (T - 1) \text{ Est } R_s$  and sum of  $X L_2 = (T - 1)L_s$ . Here  $T$  is equal to the total number of sub-groups, and all summations are over the sub-groups. However, these relationships together with the three identities given in set (I) do not provide unique estimates of  $R_1$ ,  $R_2$ ,  $L_1$  and  $L_2$ .

To get reasonably good estimates we work with  $(1 - X)L_1$  and  $L_s$  As the

sum of  $(1-X)L_i$  over all the sub-groups equals  $L_s$ , we may write the following three identities :

$$\begin{aligned} (1-x)r_1 + xr_2 &= r_s \\ (1-x)\text{est } r_1 + x\text{est } r_2 &= \text{est } r_s, \quad \text{and} \\ (1-x)l_1 + xl_2 &= l_s \end{aligned} \tag{II}$$

The parameters in set (II) are similar to those in set I except that now  $\bar{x}$ , replaces the total populations,  $(1-X)L_i$  replaces the sub-group populations, and difference between  $L_s$  and  $(1-X)L_i$  replaces the rest of the sub-group populations. It can be shown that

$$r_1 \text{ for 1981 is approximately equal to } \left(1 + \frac{x}{2x-1} \frac{S}{\text{est } r_s}\right) r_s.$$

Where  $S = (\text{est } r_1 + \text{est } r_2 - 2 \text{est } r_s)$  for 1981. Putting

$$r_1 = \left(1 + \frac{x}{2x-1} \frac{S}{\text{est } r_s}\right) r_s \tag{III}$$

we have the estimated value of  $(1-X)L_1$  in 1981 given by the relation :

$$(1-X)L_1 = (\text{Cumulated } (1-X)L_1 \text{ upto 1971}) (r_1 - 1) \tag{IV}$$

And  $\frac{(1-X)L_1}{L_s} N$  will the  $r$  give the additional population in sub-group between 1971 and 1981. The variance of the sub-group's population will be given by  $\left[N \frac{(1-X)L_1}{L_s}\right] \left[1 - \frac{(1-X)L_1}{L_s}\right]$ . The  $N$  in these two expressions is equal to growth of the total population between 1971-1981.

To fix the idea let us work out the estimated population for Andhra Pradesh in 1981, given all India populations for 1951, 1961, 1971, 1981 and Andhra Pradesh's populations for 1951, 1961 and 1971. Andhra Pradesh—an important southern Indian State—appears as the first State in Indian census tabulations by States,

### 3. Numerical Illustration

In Table 1 the initial computations for all India are presented. Upper panel of the table shows the computations with population figures, whereas the lower panel shows the computations with  $L_s$ .

TABLE 1-PARAMETERS FOR ALL INDIA

	1951	1961	1971	1981
	(1)	(2)	(3)	(4)
Pop.	361,088,091	439,234,771	548,159,652	615,184,692
Cum		800,321,862	1348,482,514	2033,667,206
R <sub>s</sub>		2.216420 <sup>a</sup>	1.684923	1.508115
Est R <sub>s</sub>		2.0	1.548822 <sup>b</sup>	1.406501
L <sub>s</sub>		.216420	.136101 <sup>c</sup>	.101614
Cum L <sub>s</sub>			.352521	.454135 <sup>d</sup>
r <sub>s</sub>			1.628874	1.288249
est r <sub>s</sub>				1.386079

$$a \quad 800,322,862 \div 361,088,091$$

$$b \quad 2 - \frac{1}{2.216420}$$

$$c \quad 1.684923 - 1.548822$$

$$d \quad .216420 + .136101 + .101614$$

Entries in the lower panel of Table 1 are common for all sub-regional estimations. Table 2 shows the estimation of parameters for the State Andhra Pradesh. Entries under 1981 are the inputs for Table 3 where the expected population for 1981 is given.

TABLE 2-PARAMETERS FOR THE STATE ANDHRA PRADESH IN 1981

	1951	1961	1971	1981
	(1)	(2)	(3)	(4)
Pop.	31,115,259	35,983,447	43,502,708	
Cum		67,098,706	110,601,414	
R <sub>I</sub>		2.156457	1.648339	
Est. R <sub>I</sub>		2.0	1.536276	
L <sub>I</sub>		.156457	.112063	
(I - X)		.086171	.083840 <sup>a</sup>	
L <sub>I</sub> (I - X)		.013482	.009395	
Cum I			.022877 <sup>b</sup>	
r <sub>1</sub>			1.696886 <sup>c</sup>	
est. r <sub>1</sub>				1.410685
Cum II		.202938	.329644 <sup>d</sup>	
r <sub>2</sub>			1.624358	
est r <sub>2</sub>				1.384372

$$a \quad 67,098,706 \div 800,322,862 \quad (\text{from Table 1 row 2})$$

$$b \quad .013482 + .009395$$

$$c \quad .022877 - .013482$$

$$d \quad .352521 \quad (\text{from Table 1 row 6}) - .022877$$

In Table 3 the footnotes explain the main computations where some of the inputs are taken from Tables 1 and 2. Row number "•" gives the estimate of  $(1 - X) L_1$  for 1981, the next row gives the proportion of  $L$ , which falls in Andhra Pradesh

TABLE 3-ESTIMATED POPULATION OF ANDHRA PRADESH IN 1981

	<i>1981</i>
<b>est. <math>r_1</math> (from row 10, Table 2)</b>	1.410682
<b>est. <math>r_2</math> (from row 13, Table 2)</b>	1.384372
<b>est. <math>r_s</math> (from row 8, Table 1)</b>	1.386079
<b><math>S</math></b>	.022899
<b><math>X</math></b>	.935104 <sup>a</sup>
<b>Estimated <math>R_1</math> (1981)</b>	1.311119 <sup>b</sup>
<b><math>(1 - X) L_1</math></b>	.007117 <sup>c</sup>
<b><math>(1 - X) L_1 \div L</math></b>	9,597.805 <sup>c</sup>
Estimated Population	53,100,513
% error	-0.84 <sup>f</sup>
Standard deviation	2988

•  $.329644$  (from row 11, Table 2)  $\div$   $.352521$  (from row 6, Table 1)

b  $\left[ 1 + \frac{0.022899 \times .935104}{(2 \times .935104 - 1) \times 1.386079} \right] 1.288249$

c  $(1.311119 - 1) .22877$  (from row 8, Table 2)

d  $.007117 \div 101614$  (from row 5, Table 1)

e  $N = 137,025,040$  growth of India's Pop. between 1971-81

f (estimated pop. - enumerated Pop.)  $100 \div$  enumerated Pop.

$(1 - X) L_1 \div L_s$  for all States taken together form a multinomial distribution, as such, mean and variance for each cell corresponding to a State can be computed. Adding the expected addition 9,597,805 to the enumerated population of 1971, we get the estimated population of Andhra Pradesh in 1981 as (9,597,805 + 43,502,708), From the row number 11 of Table 3 it can be seen that the difference between the estimated and enumerated populations is very small. The last row of Table 3 gives the standard deviation of the estimated additional population between 1971-81.

#### 4. State Populations in 1981

Applying the procedure described in Section 3, expected populations in 1981 for the 22 States as in 1971 were estimated, a sub-region labelled as "others" was added to account for the populations in the union territories. Recently

two more States were added. This shows how difficult it is to decide the pattern for the conventional "ratios" in ratio method of estimation. The present method will be applicable for the previous three censuses. The ratio method or the present method can be applied beyond 1991 as Registrar General of India is likely to provide estimated populations of the two new States for the earlier periods in 1991 census publications.

TABLE 4- ESTIMATED POPULATION BY STATE IN 1981

<i>Name</i>	<i>Estimated Population</i>	<i>% error</i>	<i>Standard deviation</i>
	(1)	(2)	(3)
1. Andhra Pradesh	53,100,513	-0.84	2988
2. Assam	19,776,554	-0.60	2156
3. Bihar	68,667,502	-1.78	3348
4. Gujarat	34,320,811	0.69	2683
5. Haryana	13,072,718	1.16	1723
6. Himachal Pradesh	4,285,915	0.12	906
7. Jammu & Kashmir	6,064,244	1.28	1197
8. Karnataka	36,462,671	-1.81	2606
9. Kerala	26,893,971	5.66	2307
10. Madhya Pradesh	53,425,276	2.39	3280
11. Maharashtra	64,135,175	2.15	3514
12. Manipur	1,443,139	1.56	608
13. Meghalaya	1,319,607	-1.21	554
14. Nagaland	774,855	-0.01	508
15. Orissa	27,540,981	4.44	2317
16. Punjab	16,527,119	-1.56	1706
17. Rajasthan	32,781,655	-4.32	2580
18. Sikkim	271,898	-14.06	249
19. Tamil Nadu	51,037,335	5.43	3022
20. Tripura	2,023,471	-1.44	682
21. Uttar Pradesh	106,672,775	-3.78	3985
22. West Bengal	55,619,731	1.90	3221
23. Others	8,967,476	-8.70	1605
India	685,184,692	0.0	—

Table 4 presents the estimated populations of 1981 along with percentage deviation from the enumerated population and standard deviation of each estimate. In three States out of the 22 States, namely, Kerala, Tamil Nadu and Sikkim, the percentage deviations from enumerated populations in 1981 were more than 5%. Kerala experienced heavy out-migration and Sikkim received large number of immigrants. Migration need not be the only cause of error. For example, Orissa showed over-estimation of 4.44%. Similarly, Tamil Nadu showed an over-estimation of 5.43%. The over-estimation in these two states can be attributed to sharper fall in fertility between 1971-81. One important feature of this method of projection is that it is not dependent on the size of the base population. For example, from Table 4 it can be seen that the error in projection for Nagaland is - 0.01% where the total population was about 775,000, whereas in the population giant Uttar Pradesh, where the enumerated population in 1981 was more than 110 million, the error of estimation is — 3.78%. The standard deviations given in the last column of Table 4 refer only to the additional population in 1971-81.

### 5. Age Distribution of the Total Population in 1981

Estimation of population by age in 1981, given the total population in 1951, 1961, 1971 and 1981; and population by age in 1951, 1961 and 1971 is relatively easy. This happens because in the absence of international migration, the proportion above age 10 in 1981 should be smaller than the proportion above age 0 in 1971. Similarly, the proportions above ages 15, 20, 25, . . . . 60 in 1981 should be respectively smaller than proportions above ages 5, 10, 15, . . . , 50 in 1971. In the notations used here this will mean that

$$(1 - X) L_1 < L_s \left( \begin{array}{l} \text{Cumulated proportions above ages 10} \\ \text{years younger in 1971} \end{array} \right).$$

And by successive subtraction the upper limit of  $(1 - X)L_i$  for each age group 10-14, 15-19, 20-24, . . . , 55-59, 60 + in 1981 can be estimated. For the remaining two age groups — 0-4 and 5-9 — the  $(1 - X)L$  values are estimated using the procedure described in Section 3.

In Table 5, column 1, the upper limit of  $(1 - X)L_i$  for each age group is given. Since the lower limit in each age class is zero, the standard deviation of estimated additional population between 1971-81 in each age group will be proportional to the corresponding entry in column 1 of Table 5. Now, if  $(1 - X)L_i$  are correctly estimated, then their sum over all ages should be equal to  $L_s$ . In other words, prorata adjustment of entries in column 1 of Table 5 such that the sum equals to  $L_s$  will amount to adjustment of the first estimates in proportion to standard deviations. Column 2 of Table 5 shows the adjusted

$(1 - X) L_1$  these values divided by  $L_S$  gives the estimated proportions in the various age groups and each ratio multiplied by  $N$  gives the estimated additional population in 1981. These entries are shown in column 3 of Table 5. Adding the entries in column 3 of Table 5 to the enumerated populations in the respective ages in 1971 will give the estimated population by age in 1981.

The estimated populations are shown in column 4 of Table 5, and the last column shows the percentage deviations from the enumerated populations by age in 1981.

TABLE 5—ESTIMATED POPULATION BY AGE IN 1981

<i>Age Group</i>	<i>(1-x) 1st estimate</i>	<i>Adjusted (1-X) L<sub>1</sub></i>	<i>N (1-X) L<sub>1</sub> ÷ L<sub>s</sub></i>	<i>Estimated Population</i>	<i>% error</i>
	<i>(1)</i>	<i>(2)</i>	<i>(3)</i>	<i>(4)</i>	<i>(5)</i>
0-4	.010969	.008743	11,789,812	91,349,328	5.84
5-9	.014899	.011876	16,014,618	98,022,090	1.64
10-14	.014748	.011755	15,851,451	84,619,285	-4.15
15-19	.015202	.012117	16,339,603	63,807,835	-3.27
20-24	.012748	.010161	13,701,965	56,803,319	-3.79
25-29	.008799	.007014	9,458,280	50,278,730	-3.87
30-34	.007990	.006369	8,588,506	44,776,923	2.36
35-39	.007567	.006032	8,134,066	41,032,369	2.44
40-44	.006708	.005347	7,210,354	35,498,338	0.67
45-49	.006099	.004861	6,554,990	29,439,773	-2.24
50-54	.005244	.004180	5,636,671	26,167,595	-0.12
55-59	.004242	.003381	4,559,231	17,387,620	2.48
60+	.012267	.009778	13,185,494	46,001,490	3.37
All Ages	.127482	.101614	137,025,041	685,184,694	0.00

From Table 5, column 5, it can be seen that error in projection is rarely more than the 4%. Only for the 0 to 4 age group it was more than 5%. The reason of course, is faster fall in fertility between 1971-81 compared to the two earlier decades 1951-61 and 1961-71. It can also be seen that as in the case of State level projections the extent of error does not depend on the size of the base population. Indeed, the minimum error is in the age group 50-54 which had about 26.2 million people in 1981 compared to 91.3 million in 0 to 4 ages.

## 6. District Level Projection

In Kerala there were 12 districts in 1981 one of which—Idduki—is a new district created after 1971. However, in 1981 census publications the Registrar General and Census Commissioner of India had provided the estimated populations of this district in the earlier censuses. Two districts, Wayned and Iddnki, are relatively small in size—having 554,026 and 971,636 persons respectively in 1981. The other 10 districts had more than 1.5 million persons each in 1971.

Using district Level data from 1951 to 1971 censuses and Kerala's total population for the censuses of 1951, 1961, 1971 and 1981 the parameter  $(1 - X) L_1 \div L_s$  was computed for each district.  $N$  or intercensal growth for Kerala in 1971-81 was 4,106,305 persons. Adding AT  $(1 - X) L_1 \div L_s$  to the enumerated population in 1971 the estimated populations for 1981 are estimated. These are shown in column 2 of Table 6, and the next column shows the percentage error on the basis of 1981 enumerated population. As in the two earlier sub-group projections error of estimation appears to be small and the level of error is not dependent on the size of the base population.

TABLE 6-ESTIMATED POPULATION BY DISTRICT IN KERALA-1981

<i>District</i>	$\frac{N(1-X)L_1}{L_s}$	<i>Estimated Population</i>	<i>% error</i>
Cannanore	521,980	2,757,809	- 1.63
Wayned	127,428	541,278	-2.30
Kozhikode	400,145	2,221,879	- 1.04
Malapporam	483,355	2,344,712	-2.41
Palghat	311,112	2,016,459	-1.37
Triclmr	429,473	2,558,270	-4.87
Ernakulum	451,654	2,615,328	- 3.16
Idduki	119,952	885,560	- 8.86
Kottayam	202,477	1,741,507	2.60
Alleppey	273,758	2,399,480	2.10
Quilon	377,008	2,789,828	- 0.85
Trivandrum	384,658	2,583,294	- 0.49
Kerala	4,088,030 <sup>a</sup>	25,455,404 <sup>a</sup>	0.00

<sup>a</sup>Deviation on from actual value or is due to founding.

## 7. All India Projection Beyond 1981

For long term projection at country level, data on total population of India from 1901 to 1981 censuses have been used. Further, it has been assumed that India's population will stop growing between 2041 and 2051.

Table 7 show a she basic input and the estimated parameters from 1901 to 1981.

From the last row of Table 7 it can be seen that only once, in 1921,  $L_s$ , was negative. This corresponds with the fall of total population between 1911 and 1921. In the subsequent years  $L_s$  increased though not uniformly, for example, between 1941 and 1951 there was a fall in  $L_s$  from 0.038875 to 0.031675. This fall can be attributed to the shift of population between India and Pakistan in that decade. Interestingly, the age distribution also shows the impact of such population movements. The population in the 0 to 14 ages showed a dip in 1951 in several States of India, The people moving to Pakistan from India were mostly Muslims. It is a historical fact that Muslims had higher fertility throughout and 10 they had relatively higher proportion of persons in the 0 to 14 ages The exchange of population between India and Pakistan resulted in apparent decrease in child population in India The sudden drop of proportion of persons from 25-29 to 30-34 ages in 1981, which can be seen from the age distribution of 1981 population, can be attributed to the smaller 0 to 4 cohort of 1951. Thus, though the quality of Indian census data is not all that good, important historical events like the effect of influenza epidemic, partition of the country etc. can be detected from the total counts as well as the age distributions.  $L_s$  continued to increase between 1951-1971 and it appears that the growth has levelled off in 1971-81. It is expected that  $L_s$ , will decline after 1981 and zero growth, or  $L_s = 0$ , will be attained some time in the next century.

To estimate the likely values for  $L_s$  corresponding to the years 1991, 2001, ..., 2041, 2051, two methods are adopted here In the first it was assumed that  $L_s$  will fall to zero in 2051 from its value .050978 in 1981 along a linear path; corresponding estimates of the total population are given in the upper panel of Table 8,

In the lower panel of Table 8 the alternative path for  $L_s$  and corresponding estimated total populations are given. For the alternative path a high degree polynomial was fitted to  $L_s$ , and  $S$  using the known values of  $L_s$ , for  $S = 1, 2, 3, 4, 5, 6, 7$  and 14 corresponding to 1921, 1931, . . . 1981 and 2051.  $L_{14}$  has been assumed to be zero. The curve fitted was taken as

$$L_s = A + \frac{B}{S} + \frac{C}{S^2} = \frac{D}{S^2} + \dots + \frac{H}{S^7} \quad (V)$$

From (V), the  $L_s$  values for 1991, 2001, . . . , 2041 were estimated by putting  $S = 8, 9, 10, 11, 12$  and 13 respectively.

TABLE 7— ENUMERATED POPULATION OF INDIA 1901 TO 1981 AND ITS PARAMETERS

<i>Popu- lation</i>	<i>1901</i>	<i>1911</i>	<i>1921</i>	<i>1931</i>	<i>1941</i>	<i>1951</i>	<i>1961</i>	<i>1971</i>	<i>1981</i>	<i>1991</i>
Popn. in ('000)	238,396	252,093	251,321	278,977	318,660	361,088	439,235	548,160	685,185	
	Popu- lation	490,490	741,810	1020,788	1339,449	1700,537	2139,772	2687,931	3373,116	
<i>R<sub>s</sub></i>		2.057455	1.512388	1.376076	1.312171	1.269580	1.258292	1.256177	1.254912	
Est. <i>R</i>			1.513963	1.338794	1.273296	1.237904	1.212338	1.205272	1.203934	1.203131
<i>L<sub>s</sub></i>			-.001574	.037282	.038875	.031675	.045954	.050905	.050978	

TABLE 8— ESTIMATED ALL INDIA POPULATION 1991 TO 2051

*Linear decline in L<sub>s</sub>*

	<i>1991</i>	<i>2001</i>	<i>2011</i>	<i>2021</i>	<i>2031</i>	<i>2041</i>	<i>2051</i>
<i>L<sub>s</sub></i>	.043695	.036413	.029131	.021849	.014567	.007285	.000003
<i>R</i>	1.246826	1.234376	1.219005	1.201508	1.182280	1.161462	1.139016
Popn.	832,573,983	915,714,756	1136,943,969	1275,211,744	1385,973,755	1451,465,248	1451,467,759

*Non-linear decline in L<sub>s</sub>*

<i>R<sub>s</sub></i>	.048296	.043526	.037020	.029106	.020106	.010318	0.0
<i>R<sub>s</sub></i>	1.251427	1.244438	1.233444	1.218368	1.199336	1.176523	1.150038
Popn.	848,093,689	1031,825,050	1226,291,585	1414,877,109	1573,598,300	1671,287,600	1671,285,184

Since Est , for 1991 is known, once  $L_s$  for 1991 is assigned a value,  $R_s$  for 1991 is known. Having estimated  $R_s$  for 1991 the Est  $R_s$  for 2001 becomes known, to which when  $L_s$  for that year is added the  $R_s$  for 2001 becomes known. Following this simple technique  $R_s$  for each census year is estimated and the estimated population in any census year is equal to

$$(R_s - 1) (\text{Cumulated population upto the previous census}) \quad (\text{VI})$$

From Table 8 it is clear that linear decline of  $L_s$  to zero by 2051 will imply a total population smaller by nearly 200 million compared to the projection based on the observed pattern of  $L_s$  from 1921 to 1981 and to zero in 2051. It may be impossible to predict the exact pattern which  $L_s$  will follow, but the fact remains that India has to plan for more than double its 1981 population in the next 65 to 70 years.

### Discussion

This expository paper shows that a simple mathematical model can be used to predict both total and sub-regional populations. However, it is felt that for predicting the total population the method based on the net reproduction rate should be preferred as it has several other advantages (Pichat and Taleb, 1970). For sub-group or sub-regional projections the method described here gives fairly reliable estimate irrespective of the size of the base population. Further, as the standard deviations are usually much smaller than the deviations from the actual, the implication is that the deviations are due to departure from past trend and not due to error of estimation. In developing countries therefore, projection by component method will become more difficult as demographic transition intensifies.

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